Slide rules, used by engineers in the 19th and early 20th centuries, employ the principle of logarithms for performing complicate calculations.

The definition and three properties of logarithms are summarized in this box.

DEFINITION AND PROPERTIES: Base-10 Logarithms

Definition

$\log x = y$ if and only if $10^y = x$

Verbally: $\log x$ is the exponent in the power of 10 that gives x

Properties

- Log of a Product:

  $\log xy = \log x + \log y$

  Verbally: The log of a product equals the sum of the logs of the factors.

- Log of a Quotient:

  $\log \frac{x}{y} = \log x - \log y$

  Verbally: The log of a quotient equals the log of the numerator minus the log of the denominator.

- Log of a Power:

  $\log x^y = y \log x$

  Verbally: The log of a power equals the exponent times the log of the base.

The reason for the name **logarithm** is historical. Before there were calculators, base-10 logarithms, calculated approximately using infinite series, were recorded in tables. Products with many factors, such as

$$(357)(4.367)(22.4)(3.142)$$

could then be calculated by adding their logarithms (exponents) column-wise in one step rather than by tediously multiplying several pairs of numbers. Englishman Henry Briggs (1561–1630) and Scotsman John Napier (1550–1616) are credited with inventing this "**log**ical way to do **arithm**etic" that you will explore in Problem 47. The word *logarithm* actually comes from the Greek words *logos,* which here means "ratio," and *arithmos*, which means "number."

The most important thing to remember about logarithms is this:

A logarithm is an exponent.

Examples 1 and 2 show you how to verify that a logarithm is an exponent.

EXAMPLE 1 ➤ Find x if $\log 10^{3.721} = x$. Verify your solution numerically.

SOLUTION By definition, the logarithm is the exponent of 10. So $x = 3.721$.

CHECK $10^{3.721} = 5260.1726\ldots$ By calculator. Do not round.

$\log 5260.1726\ldots = 3.721$

which checks.